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# Revisiting Mixed Traffic Fundamental Diagram under Vehicle Sequence Autocorrelation

Session: Intelligent and Connected Vehicles (1)

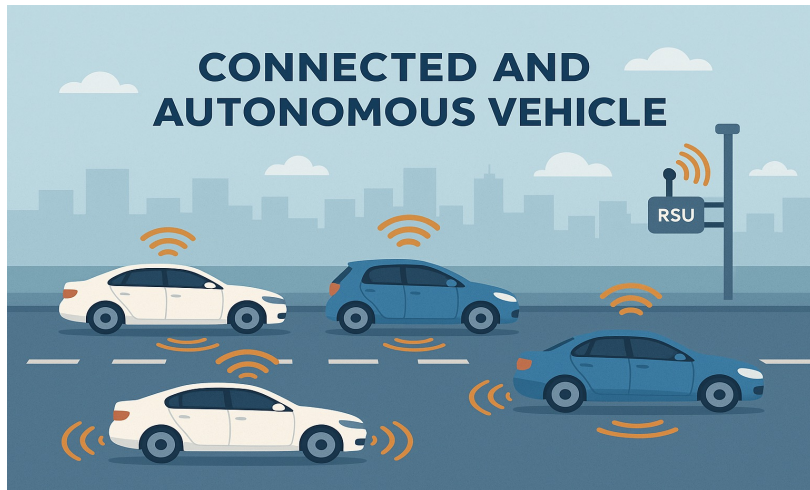
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09 Aug 2025

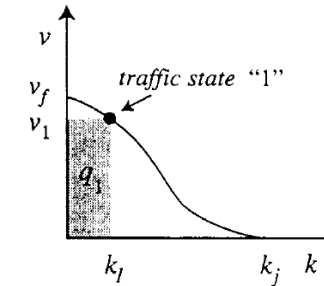
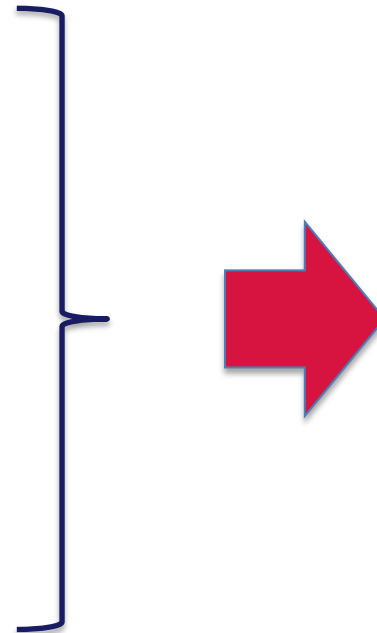


# Traffic Flow Theory in the Mixed Traffic Era

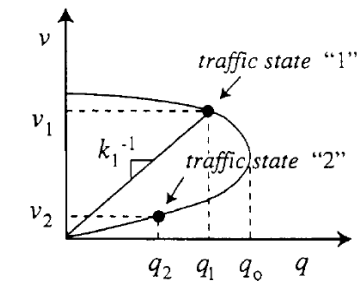


## Mixed Traffic Characteristics

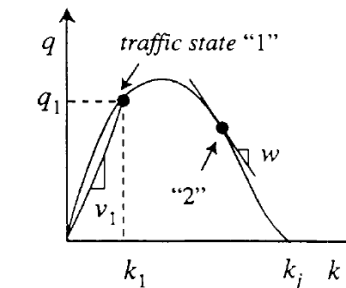
- Penetration Rate
- Platooning Intensity
- Platoon Size
- ...



(a)



(b)

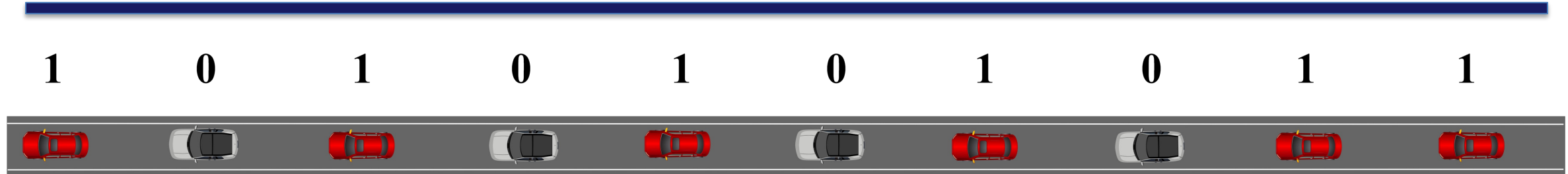


(c)

## Traffic Flow Theory

- Capacity
- Jam Density
- Fundamental Diagram
- ...

# Typical Mixed Traffic Modeling

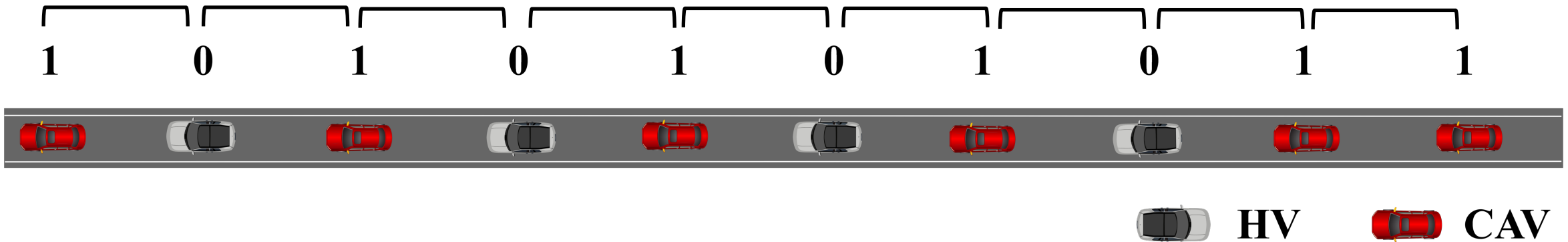


 **Human-driven Vehicle (HV)**       **Connected and Autonomous Vehicle (CAV)**

	HV	CAV
Features	<ul style="list-style-type: none"> <li>Human-driven</li> </ul>	<ul style="list-style-type: none"> <li>Sensing surroundings</li> <li>Autonomous decision making</li> <li>Inter-vehicle Communication</li> </ul>
Label	"0"	"1"
Penetration Rate	Proportion of HVs $P_0$	Proportion of CAVs $P_1$

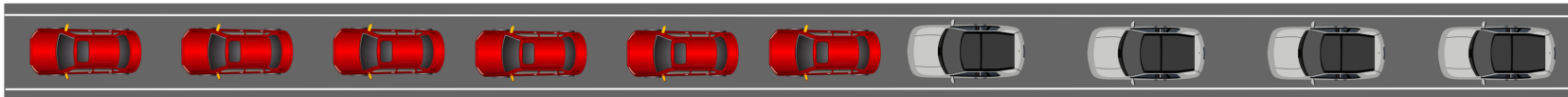
# Insights from 1<sup>st</sup> order Sequence Autocorrelation

- 1st order Sequence Autocorrelation (ACF):
  - **Relationship** between a variable's present value and its lagged values

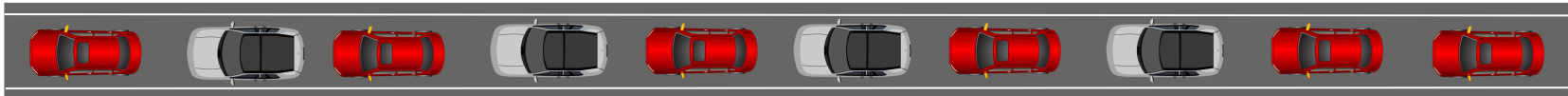


- Vehicle Sequence: Similarity of 2 consecutive vehicle types.

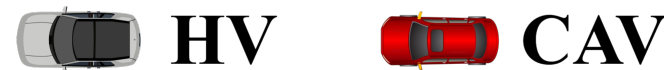
# Insights from 1<sup>st</sup> order Sequence Autocorrelation



(a) High ACF Value



(b) Low ACF Value



1<sup>st</sup> order Sequence Autocorrelation reflects the degree of clustering strength of CAVs.

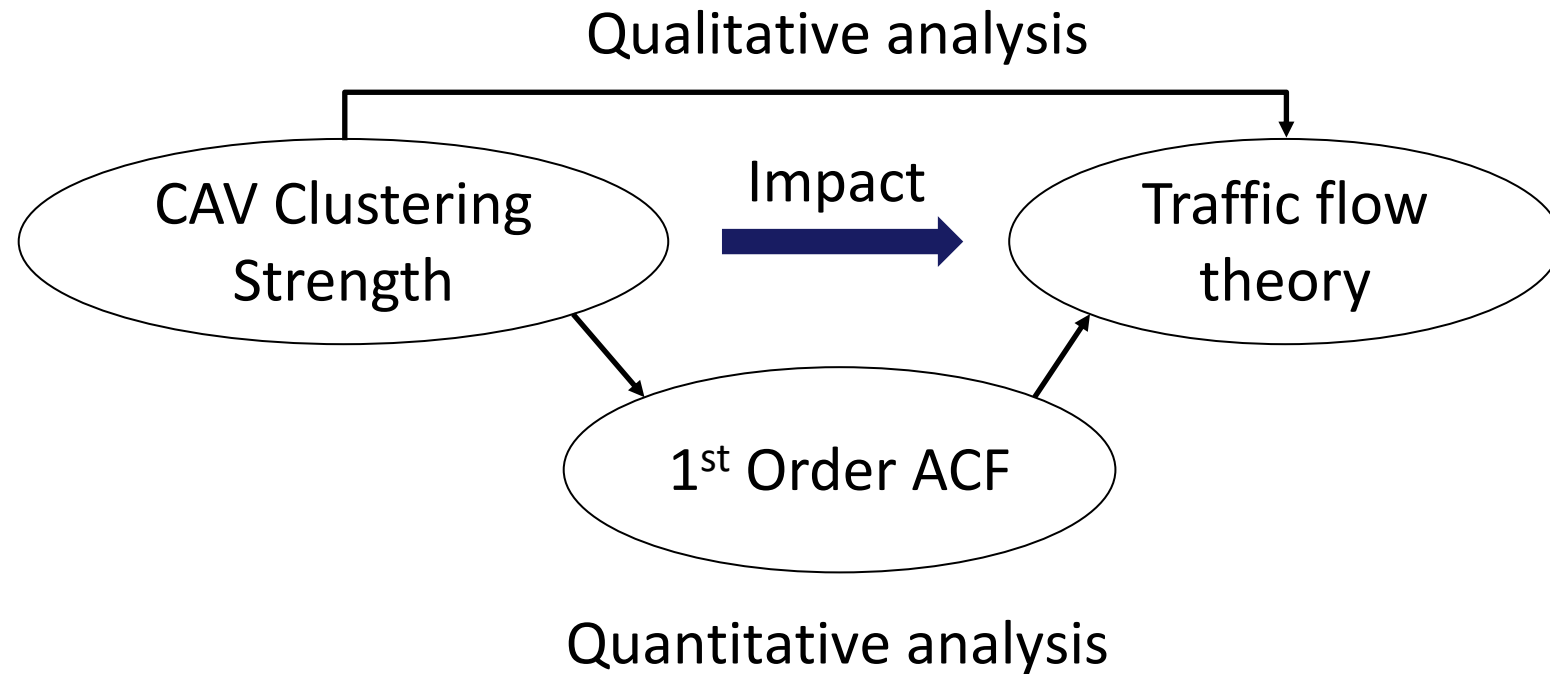
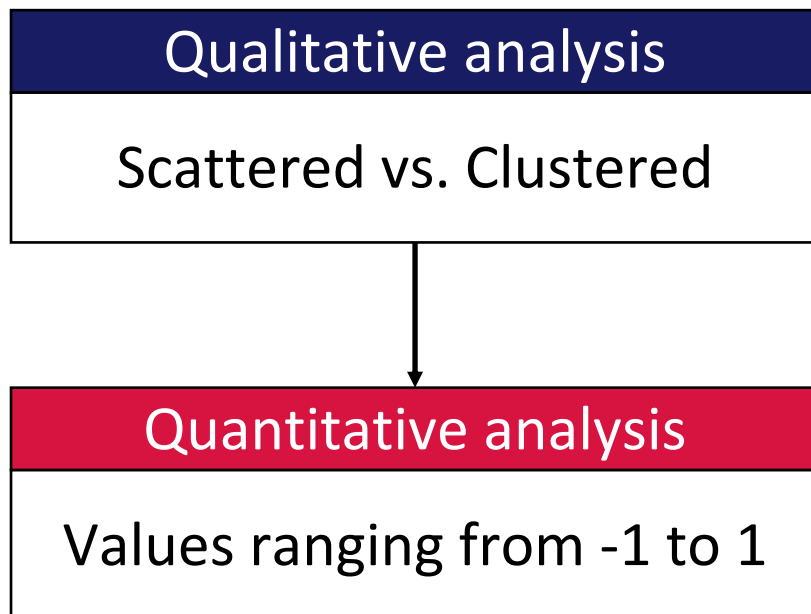
- Higher ACF -> Clustered
- Lower ACF -> Scattered

## Practically:

- Spatial clustering could provide more chances and positions for CAV inter-vehicle communications.

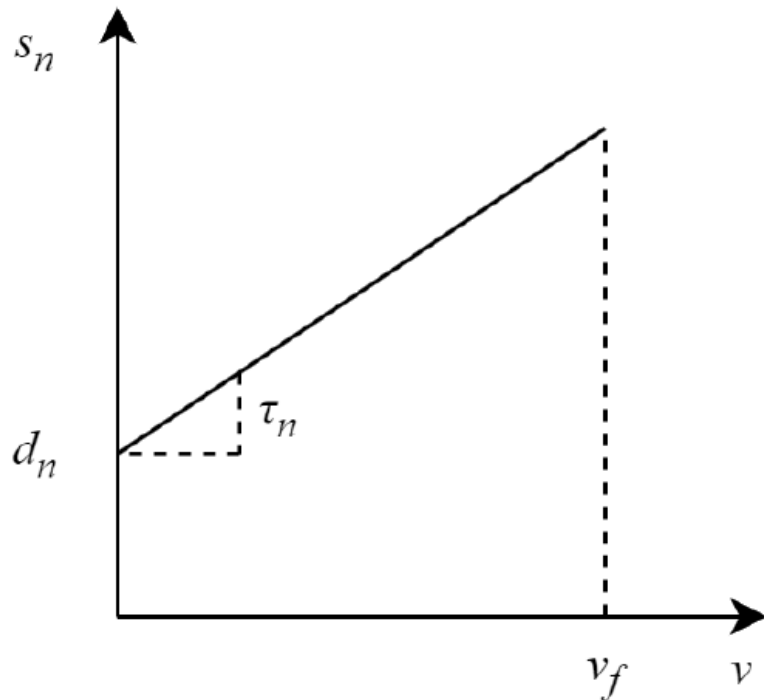
# Insights from 1<sup>st</sup> order Sequence Autocorrelation

- An new indicator of mixed traffic to quantify the **CAV clustering strength**.

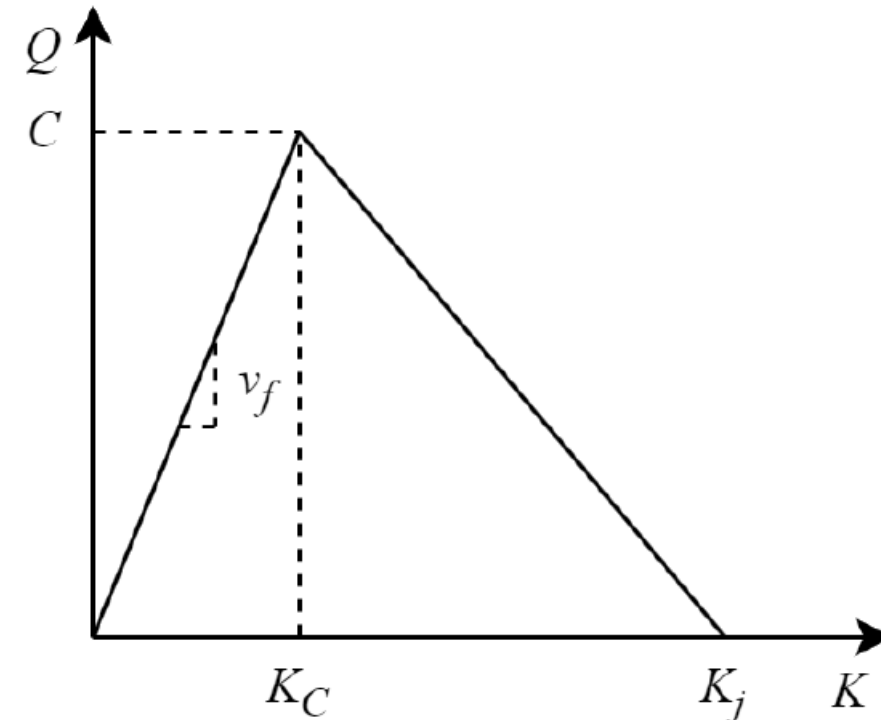


# Linear spacing-speed relation and triangular FD

- Under stationary and steady flow:

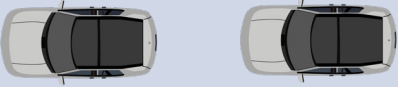

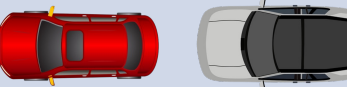
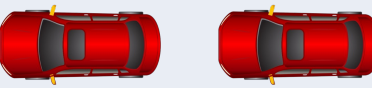


$$\bar{s} = \bar{d} + v\bar{\tau}$$



$$Q(K) = \begin{cases} v_f K, & 0 < K < K_C \quad \text{free flow branch} \\ \frac{1}{\bar{\tau}} - \frac{\bar{d}}{\bar{\tau}} K, & K_C \leq K < K_j \quad \text{congested branch} \end{cases}$$

## Estimated value of $\bar{d}$ and $\bar{\tau}$

Scenarios	Description	Digitalization
	An HV being followed by another HV	00
	An HV being followed by a CAV	01
	A CAV being followed by an HV	10
	A CAV being followed by another CAV	11

$$\bar{d} = P_{00}d_{00} + P_{01}d_{01} + P_{10}d_{10} + P_{11}d_{11}$$

$$\bar{\tau} = P_{00}\tau_{00} + P_{01}\tau_{01} + P_{10}\tau_{10} + P_{11}\tau_{11}$$

# Estimated value of $\bar{d}$ and $\bar{\tau}$

Range of  $d_{sr}$  and  $\tau_{sr}$  in the existing research

Space Displacement	Range	Time displacement	Range
$d_{00}$	[6.06,10.00]	$\tau_{00}$	[1.00, 2.00]s
$d_{01}$	[6.10,10.00]	$\tau_{01}$	[0.50, 1.50]s
$d_{10}$	[6.10,10.00]	$\tau_{10}$	[1.00, 1.80]s
$d_{11}$	[4.50,15.00]	$\tau_{11}$	[0.60, 1.00]s

1. Treiber, M., Hennecke, A. and Helbing, D. (2000) "Congested traffic states in empirical observations and microscopic simulations", Phys. Rev. E.
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3. Treiber, M. and Kesting, A. (2013) "Traffic Flow Dynamics", Springer.
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5. Levin, M. W. and Boyles, S. D. (2016) "A Multiclass Cell Transmission Model for Shared Human and Autonomous Vehicle Roads", Transp. Res. C.
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## Estimated value of $\bar{d}$ and $\bar{\tau}$

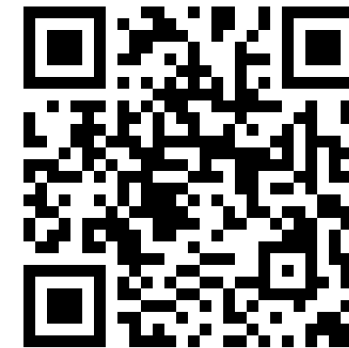
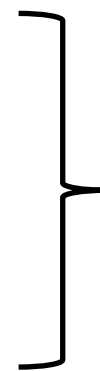
Space Displacement	Value	Time displacement	Value
$d_{00}$	7 m	$\tau_{00}$	1.60 s
$d_{01}$	7 m	$\tau_{01}$	1.10 s
$d_{10}$	7 m	$\tau_{10}$	1.50 s
$d_{11}$	5 m	$\tau_{11}$	0.60 s

$$P_{11} = P_1 [\rho(1 - P_1) + P_1]$$

$$P_{10} = P_1(1 - P_1)(1 - \rho)$$

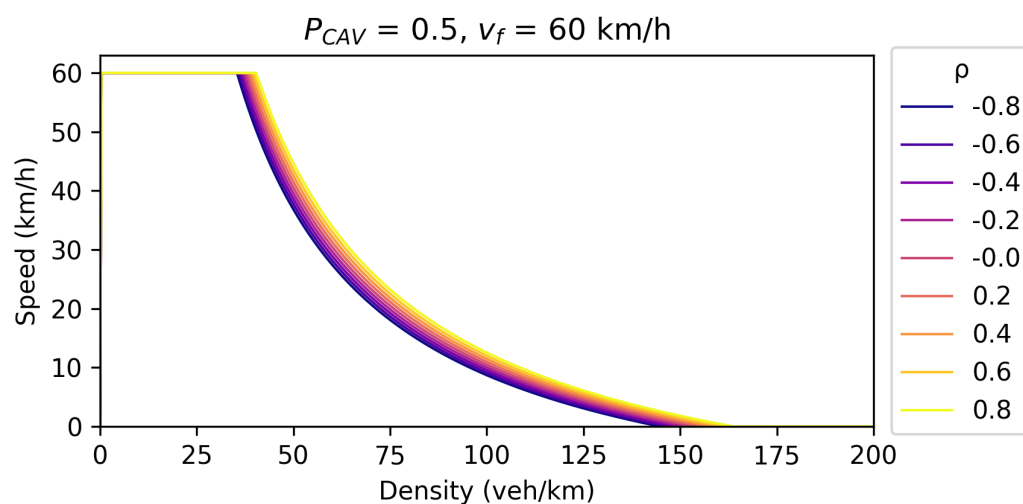
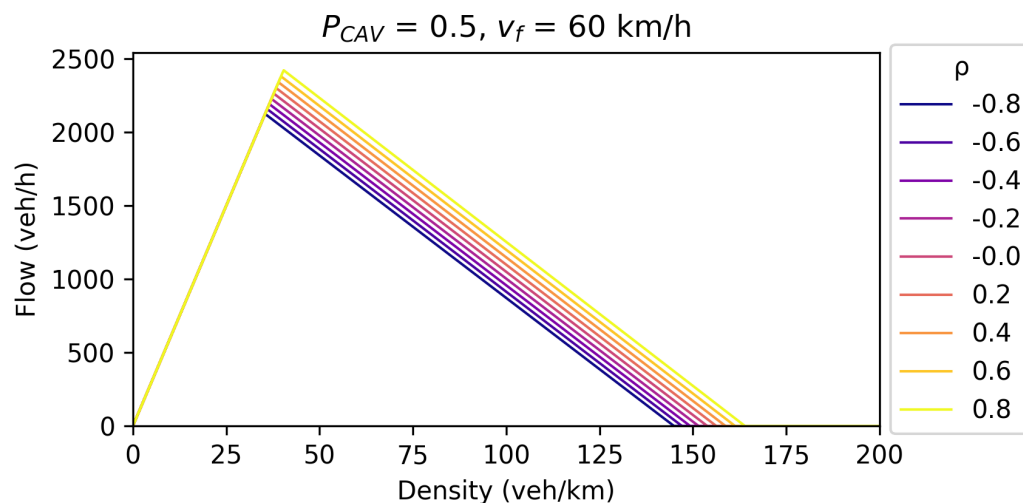
$$P_{01} = (1 - P_1)P_1(1 - \rho)$$

$$P_{00} = (1 - P_1)(\rho P_1 + 1 - P_1)$$



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# Mixed Traffic Fundamental Diagram



$\rho$	Jam density (veh/km)	Critical density (veh/km)	Capacity (veh/h)
-0.8	144.928	35.419	2125.148
-0.6	147.059	35.971	2158.273
-0.4	149.254	36.541	2192.448
-0.2	151.515	37.129	2227.723
0	153.846	37.736	2264.151
0.2	156.250	38.363	2301.790
0.4	158.73	39.012	2340.702
0.6	161.29	39.683	2380.952
0.8	163.934	40.377	2422.611

# Conclusions

1. Sequence ACF could be used for quantifying the degree of CAV clustering strength
  - Higher value of ACF indicates that CAVs are clustering together
  - The introduction of ACF promotes the quantitative analysis on the impact of clustering strength in mixed traffic environment
2. Under the conditions in this study, a higher ACF could increase the traffic efficiency
  - A higher ACF increases the mixed traffic capacity and jam density
  - A higher ACF maintains the free flow speed when density is relatively high
  - In congested branch, higher ACF increase the flow and speed for a same density.
  - Given that ACF is an indicator of clustering strength, these findings are insightful for controlling CAV clustering to tune the value of ACF on the road.
3. This research utilizes simplified equations and triangular FD under the stationary and steady condition, more complex scenarios, including nonlinear effects and dynamic terms, could be discussed in the future



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# Thank you

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Welcome the feedback to make us work better.

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